Analysis of Localization Algorithms for Sensor Networks

Jana van Greunen

Outline

- Introduction
- Algorithm overview
- Comparison & evaluation
- Extensions:
  - Real-time error estimation
  - Low-energy computation
  - Results
- Conclusion

Introduction

- Sensor network consists of many small, cheap, self-sustaining, densely deployed sensor nodes.
- Applications require that the nodes in the network are aware of their geographic location.
- Too expensive to use GPS on every node
- Thus need algorithms to compute each node's position using node-to-node range measurements and information from a few reference nodes
- Range measurements are made between each node and its neighbors within a circular area

Introduction cont'

- Measurements are made using TOA or RSSI – both methods are error prone
- A good localization algorithm should:
  - Be tolerant to range errors
  - Scale with network
  - Minimize communication & computation energy spent
  - Converge rapidly and accurately
  - Perform well across network topologies
  - Provide a measure of error

Algorithm overview

- Centralized LP (Doherty et al.)
  - Key assumption: if two nodes can communicate with each other, they must lie within the communication radius R of each other.
  - Mathematically = 2-norm constraint on the node positions

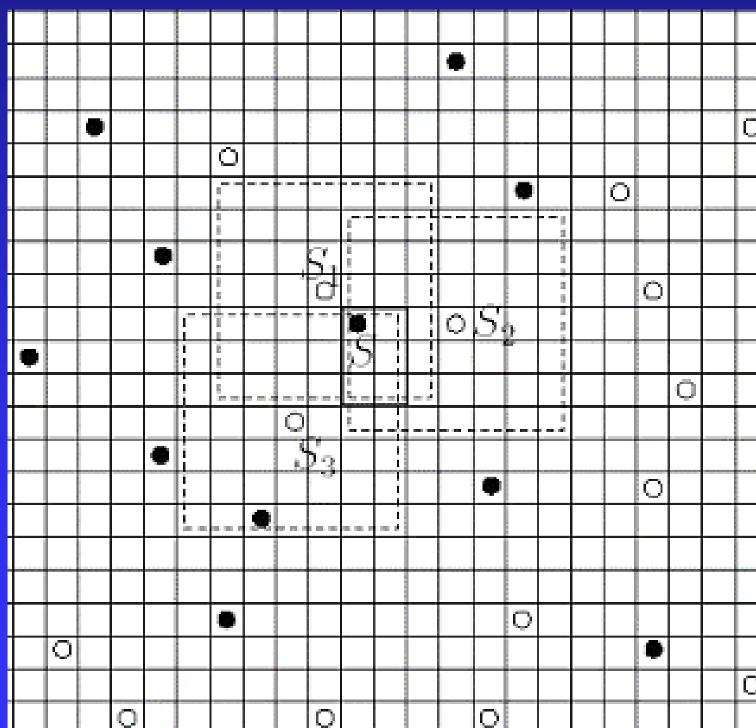
$$\|a-b\|_2 < R \Rightarrow \begin{bmatrix} I_2 R & a-b \\ (a-b)^T & R \end{bmatrix} \geq 0 \quad (xR)$$

  - Combine local connectivity-induced constraints to solve:

$$\begin{aligned} &Minimize: c^T x \\ &Subject\ to: Ax < b \end{aligned}$$

Rectangular intersection (Simič)

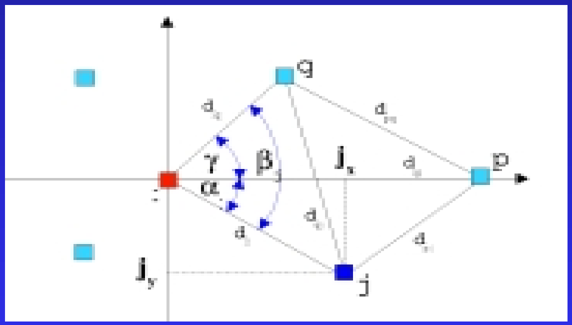
- Partition space into cubes/squares (cells)
- Communication area is a square (#cells)

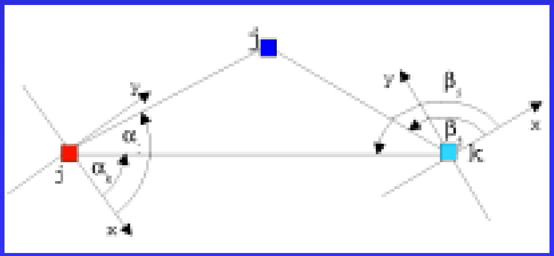


- At every unknown node:
  Step A: Gather positions of one-hop neighbors with known positions
  Step B: Compute estimated position via minimum rectangular intersection

Network Coordinate (Capkun)

- 2 phase algorithm
  - Local coordinates
    - Each node *j* measures distances to its one-hop neighbors and their distances from each other
    - Place one-hop neighbors in local coordinate system (trigonometry)





  - Global coordinates
    - Align local coordinates

DV-Hop (Niculescu et al.)

- Use ‘average’ distance to prevent error propagation
- Known nodes flood ‘hops’ and position through network
- Each unknown node stores the position and hop-distance (# hops*average distance) from all the known nodes
- When an unknown node has its hop-distance from more than three non-colinear known nodes it can compute its position via triangulation (solve Ax=b)
- This algorithm works well when topology is regular

Start-up & refinement (Savarese)

- 2-phase
  - Initial position estimate
    - DV-Hop
  - Refinement
    - Nodes try to improve their position estimates by iteratively measuring the distances to one-hop neighbors and then performing weighted maximum likelihood triangulation.
    - All unknown nodes start with a weight of 0.1
    - Known nodes have weight 1.0
    - After each position update the weight of the node is set to the average weight of all its neighbors

Comparison

	Cetralized LP	*Rectangular intersection*	*Network Coordinate*	*DV-Hop*	*Start-up & refinement*
Scalable	No	Yes	No	Yes	Yes
Energy efficient	No	Yes	Moderately	Yes	Moderately
Accuracy	Good	Spotty	Poor	Medium	Fair
Speed of convergence	Depends on network size	Fast	Depends on network size	Medium	Medium
Tolerant to range error	No	Yes	No	Yes	Moderately

Note: no algorithm provides a measure of final position error

Extensions

- Real time error estimate for centralized algorithm (Patwari et al.)
  - The Cramer-Rao Bound: lower bound on the covariance matrix of any unbiased estimator θ.
  - Mathematical formulation :

$$Var(\hat{\theta}) \geq [F^{-1}(\theta)]_{ii}$$

$$[F(\theta)]_{ij} = -E[\frac{\partial^2 \ln p(x;\theta)}{\partial \theta_i \partial \theta_j}]$$

p(x;θ) is the conditional density function

  - **Assumptions:**
    - dij's are Gaussian distributed & independent for all i,j.
    - The variance σ_d associated with dij is independent of |dij| and is the same for all measurements in the network.

Cramer-Rao bound cont'

- Using these assumptions, $p(x;\theta)_{ii}$ is the multiplied densities of all distance measurements (Gaussian with mean dij and variance σ_d)

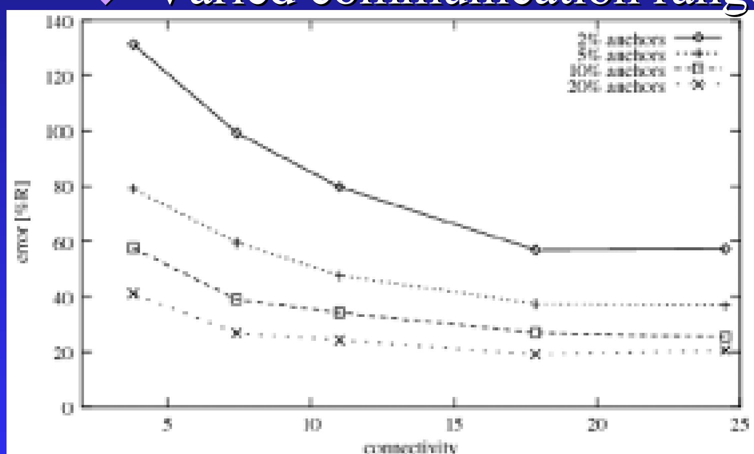
- Extend this to the distributed case:
  - Each time unknown node i estimates its position it calculates its own Cramer-Rao bound
  - However, information from unknown neighbors are used – thus need to account for the uncertainty in their positions
  - Solution: for each measurement dij increase σ_d to $\sigma_{d\,+}\,\sigma_j$

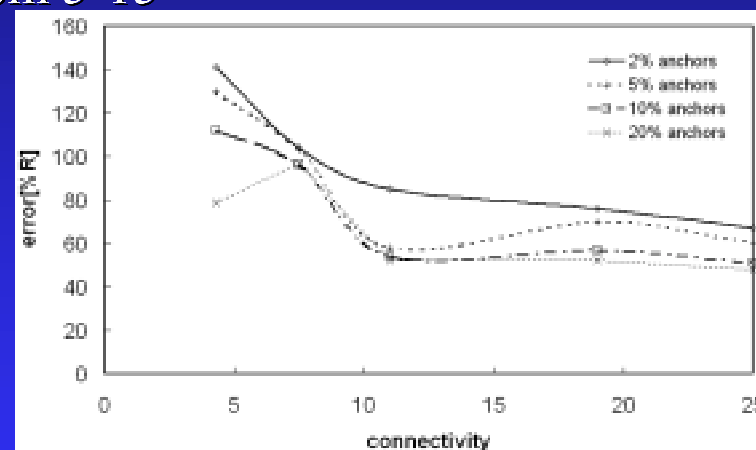
Low-power computation

- Idea: Create a low-computation, distributed and accurate algorithm.
- Combine computation of rectangular intersection with accuracy of start-up & refinement
- Replace least-squares triangulation with rectangular intersection in the start-up and refinement
- Computation savings:multiplies=>comparisons
- Also can have simpler hardware, because there are no multiplications

Results

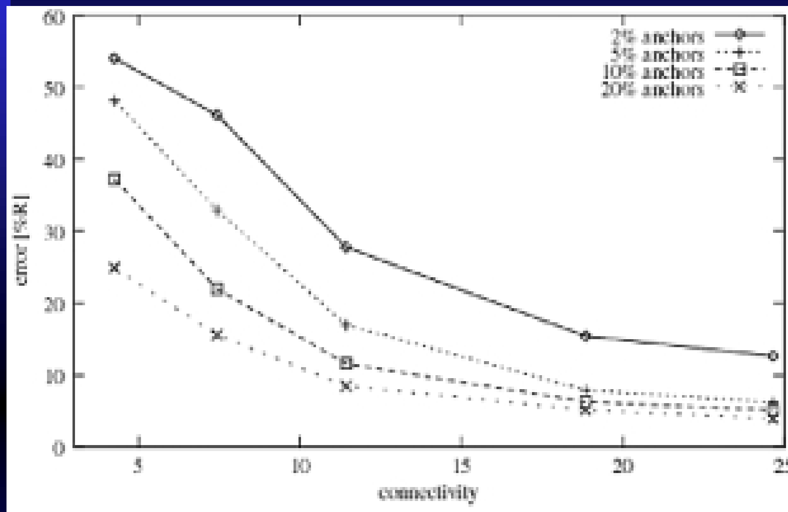
- Simulation:
  - OMNET++ (network simulation package)
  - Randomly placed 400 nodes in 100*100 rectangular area.
  - Varied communication range from 5-15

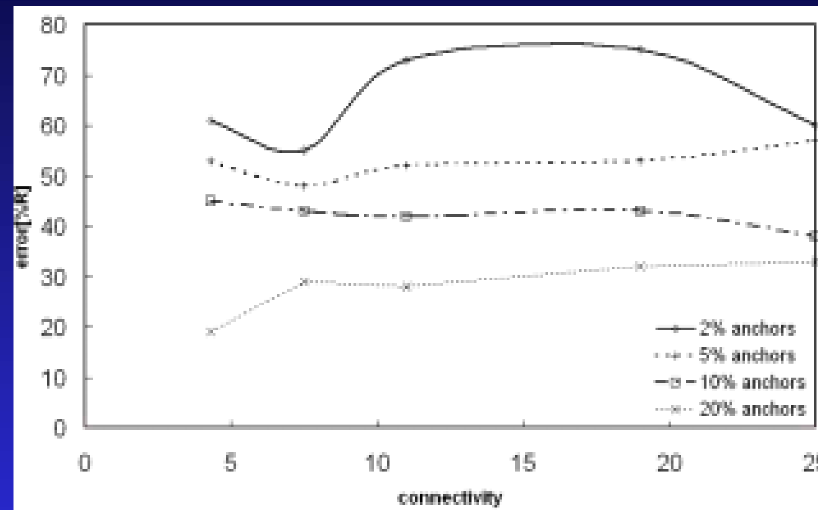




- Anchors & connectivity still => better accuracy
- On average low-power computation performs two times worse than triangulation

Results cont'





- For low connectivity – roughly the same error
- High connectivity much poorer performance
- More anchors still => better accuracy
- Connectivity does not make big difference anymore
- This is because weights are not used and correlated errors prevent convergence

Future work

- Look at non-homogenous networks, perhaps some nodes should/are more equipped to do more computation
- Incorporate a few long distance measurements to collapse error

Conclusion

- Need a scalable, energy efficient solution
- There are many different existing localization algorithms
- Performance of distributed algorithms are heavily dependent on underlying network topology
- Developed an estimate of error in the position
- Trade-off between energy and accuracy
- The low-power start-up and refinement performs 2-3 times worse than the normal algorithm